REINFORCED CONCRETE-I

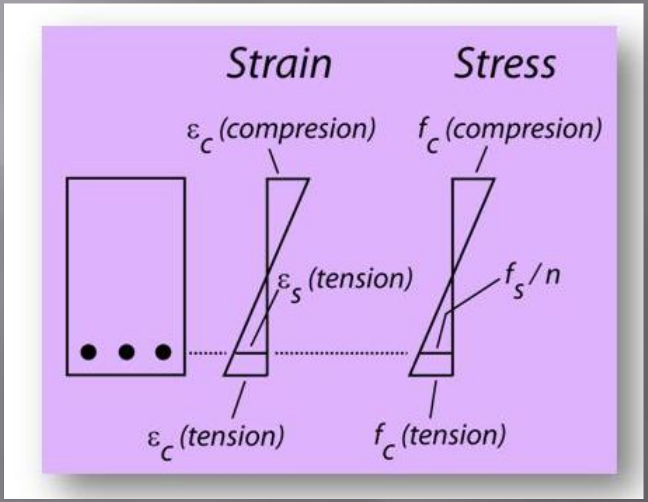
(Flexural Analysis of Beams)

Behavior stages in an RC beam

- The uncracked concrete stage
- The concrete cracked – elastic stresses stage
- The Ultimate – strength stage.

Uncracked Concrete Stage

- At small loads when the tensile stresses are less than the modulus of rupture, the entire cross section of the beam resists bending.
- Compression develops on one side and tension on the other.

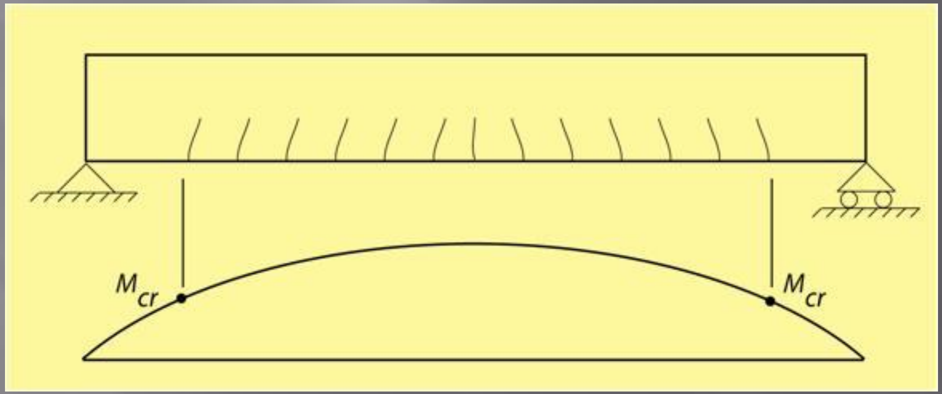


$$\varepsilon_c = \varepsilon_s \Rightarrow \frac{f_c}{E_c} = \frac{f_s}{E_s} \Rightarrow f_c = f_s \frac{E_c}{E_s}$$

$$\Rightarrow f_c = \frac{f_s}{n} \qquad \left(\because n = \frac{E_s}{E_c} \right)$$

Concrete-Cracked – Elastic Stresses Stage

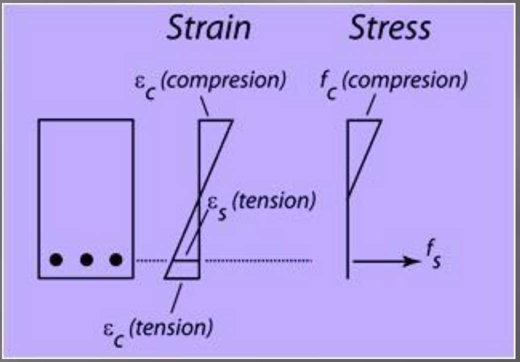
- As the load is increased after the modulus of rupture of the concrete is exceeded, cracks begin to develop in the bottom of the beam.
- Cracking Moment: ***The moment at which the tensile stress in the bottom of the beam equals the modulus of rupture (i.e. when the cracks begin to form) is referred to as the cracking moment, M_{cr}.***



- *As the load is further increased, these cracks quickly spread up to the vicinity of the neutral axis, and then the neutral axis begins to move upward.*
- *Note: The cracks occur at those places along the beam where the actual moment is greater than the cracking moment.*

Concrete-Cracked – Elastic Stresses Stage (contd.)

- As the concrete in the cracked zone can not resist tensile stresses – the steel must do it.
- In this stage the compressive stresses vary linearly with the distance from the neutral axis or as a straight line.

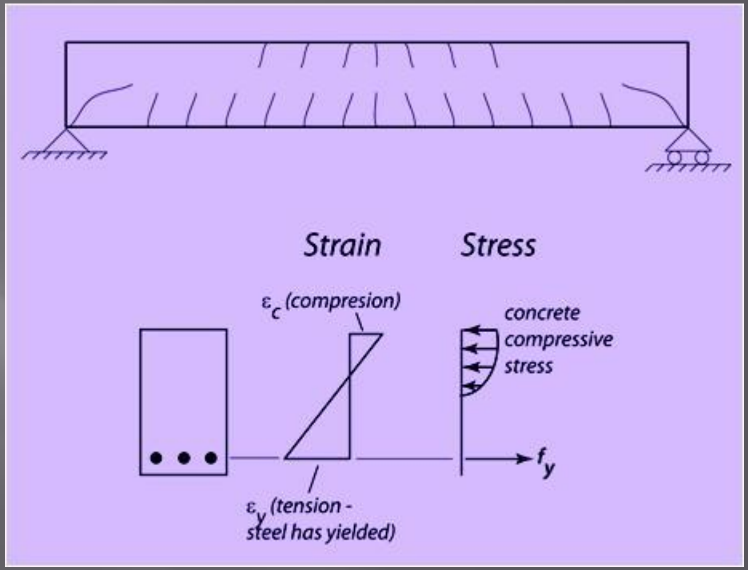


Concrete-Cracked – Elastic Stresses Stage (contd.)

- The straight-line stress-strain variation normally occurs in reinforced concrete beams under normal service-load conditions because at these loads the concrete stresses are generally less than $0.5 f_c^{'}$.
- To compute the concrete and steel stresses in this range, the transformed-area method is used.
- **Service or Working Loads:** ***The service or working loads are the loads that are assumed to actually occur when a structure is in service or use.***
- Under working loads, moments develop which are considerably larger than the cracking moments. Obviously the tensile side of the beam will be cracked.

Beam Failure- Ultimate Strength Stage

- As the load is increased further so that the compressive stresses are greater than one-half of f_c', the tensile cracks and neutral axis move further upward (for positive moment).
- The concrete compressive stresses begin to change appreciably from a straight line and reinforcing bars yield.



Bending stress in an Uncracked Beam

- The area of steel is quite small (usually 2% or less), and its effect on the beam properties is negligible as long as the beam is uncracked.
- An approximate calculation of the bending stress in such a beam can be obtained based on the gross properties of the beam's cross section.
- The stress in the concrete at a distance *y* from the neutral axis of the cross section can be determined from the following flexure formula:

$$f = \frac{My}{I_g}$$

M = Bending moment $\leq M_{cr}$ of the section

I_g = Gross moment of inertia of the cross section.

Cracking Moment

$$f = \frac{My}{I_g}$$

M = Bending moment

I_g = Gross moment of inertia of the cross section.

Substitute

$y = y_t$ = distance from the centroidal axis to its extreme fiber in tension

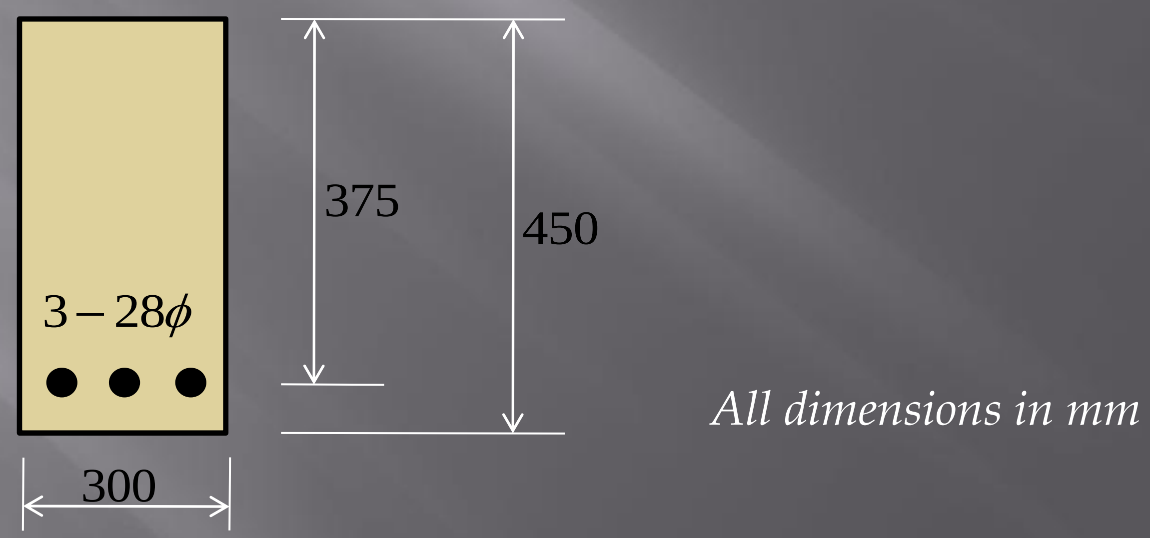
$f = f_r$ = Modulus of rupture $= 0.7\sqrt{f_c^{'}}$ (ACI section 9.5.2.3); for normal concrete.

$$\therefore f_r = \frac{M_{cr} y_t}{I_g} \Rightarrow M_{cr} == \frac{f_r I_g}{y_t}$$

Example

(a) Assuming the concrete is uncracked, compute the bending stresses in the extreme fibers of the beam of figure given below for a bending moment of 34 kN.m. The normal weight concrete has an f_c' of 30 MPa.

(b) Determine the cracking moment of the section.



(a) Bending Stresses:

$$I_g = \frac{1}{12}bh^3 = \frac{1}{12}\times 300\times 450^3 = 2.28\times 10^9 \text{ mm}^4$$

$$f = \frac{My}{I_g} = \frac{(34\times 10^6)\times 225}{2.28\times 10^9} = 3.35 \text{ MPa}$$

$$\text{Modulus of rupture } f_r = 0.7\sqrt{f_c^{'}} = 0.7\times 1.0\times\sqrt{30} = 3.83 \text{ MPa}$$

$$\because f < f_r \Rightarrow \text{The section is not to have cracked.}$$

An uncracked beam is assumed to be homogeneous with neutral axis passing through the centroid of the beam section.

(b) Cracking Moment (M_{cr}):

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.83\times 2.28\times 10^9}{225} = 38.8\times 10^6 \text{ N.mm}$$

$$\Rightarrow M_{cr} = 38.8 \text{ kN.m}$$